A SIMULATION MODEL FOR THE ANALYSIS OF A PROPOSED MINE SYSTEM

by

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THESIS

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ABSTRACT

This paper is a response to a proposal before the Mine Advisory Committee that there be an analytically supported development of a modern explosive mining capability for the U. S. Navy. A proposed system is described, and a model is developed to provide the means to evaluate the system in comparison with other alternatives.

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I. THE SYSTEM

A. SYSTEM DESCRIPTION

The system modeled in this study consisted of a pair of passive underwater bearing sensors and their associated computing and weapon delivery units. The sensors were assumed to be able to communicate their observed bearing information to the computing unit, and the computed firing solution was assumed to be fed to the weapon until launch.

In particular, the sensors were viewed as simple three-dimensional arrays of hydrophones, mounted in stabilized housings to observe horizontal bearing and verticle angle with respect to a consistent reference whose position is assumed to be errorless. The hydrophones should have a detection range for quiet targets approximately equal in magnitude to the sensor spacing. Range computations were envisioned as being entirely analog, although limited computational requirements might permit an inexpensive digital system to be used. The weapon itself was treated as a relatively high-speed, straight-running torpedo or rocket-propelled mine, programmed at launch to detonate after some run time. The sensors could communicate by radio with transmitting antennas floating at the surface if short term use were expected so that possible sweeping by an enemy need not be considered. However, the transmitting antenna systems at the surface would experience drift

with respect to the sensors which could make positioning of the sensors difficult. If one wished to keep the system flexible enough for deep water use, and not anchor the sensors, and if one also wished to provide some protection against sweeping, one might consider either a buoyed wire communication system to link the sensors, which is at best cumbersome to deploy, or a sonar communication system between the sensors, the feasibility of which is discussed below.

For the purpose of modeling the system, the sensor pairs are viewed as slave and master. The slave sensor obtains bearing information from the target and transmits the information to the master. The master uses the slave's bearing information and the slave's relative position to calculate target position and movement. The weapon is mounted at the master, and is programmed either by trainable mounting or by preset gyro to the most recently computed launch bearing and azimuth. The sensor pairs may be extended in a chain, of course, and the pair under consideration be representative of the solutions encountered anywhere in the chain. Then, deployed from ships, submarines or aircraft, the system represents a barrier to attack submarines, merchant shipping, perhaps even to high-speed non-metal construction surface attack craft. Safing and command and control capabilities are possible but are not within the scope of this paper.

B. PRELIMINARY FEASIBILITY CONSIDERATIONS

A preliminary look at system feasibility required that the sensors, communication system, computer, weapon, and method of deployment

be within the state of the art and that the system be relatively inexpensive. The sensors, weapons and method of deployment described are already in existence; their refinement and interfacings are a function of the particular system capabilities desired.

As indicated above, a wire link between sensors is cumbersome, both because it must be weightless so as not to drag the sensors together, and because it is unwieldy to deploy from surface craft, and nearly impossible by air. Also, the wire is good only for communicating target bearing information, not locating the slave sensor relative to the master. Furthermore, the wire may be broken, and sweeping operations need only cut the wire. Hence, there are tradeoffs with a sonar communicating system. These tradeoffs are not within the scope of this paper, but a few observations about underwater communication are in order.

Laser communications are possible, but at the state of the art described in Reference 5 must be rejected for our uses. Laser communication ranges are very dependent on sea conditions, and at best the ranges are too short for the project considered here. Even with longer ranges, lateral scattering of the beam makes countermeasures easy to develop.

Sound communication would require a source level of 50 to 500 watts or more depending primarily on the directivity of the source, at frequencies of 10 kHz to 100 kHz (see Appendix A). Since these are peak power requirements, the source levels are probably feasible, and

they do provide a reasonable base leg for triangulation at one to one and one-half miles. Of course the option remains to use a lower frequency to reduce absorption, and the long acquisition ranges of our extant homing torpedoes certify that the desired ranges are within the state of the art. It may be further observed that the proposed range of operating frequencies is high enough to provide good bearing accuracy in a phase-comparison system for positioning the slave relative to the master. A 100 kHz wave has a wavelength on the order of .045 feet; a low-frequency wave of even 10 kHz has a wavelength on the order of .45 feet; so that the front on the discriminating hydrophone array can be well under a foot across.

These observations demonstrate another limitation of the system: the power requirements are highly range dependent, and drift of the sensors relative to each other may put them quickly out of range (assuming the system is floating, not anchored). This problem promotes the concept of a simple propulsion system to drive the sensors laterally to offset drift. As bearing and range from master to slave are calculated in the computer no additional calculations are necessary. Since only the slaves are envisioned as being fitted with transmitters, the masters could be self-propelling to conform to the drift of the slaves. Of course, without involved coordination of the chain of sensor pairs, the chain segments might soon separate in any case. This problem might be solved through studies of drifting submarine bodies, by employing linked communication throughout the sensor chains, or by laying independent segments of slave-master-slave.

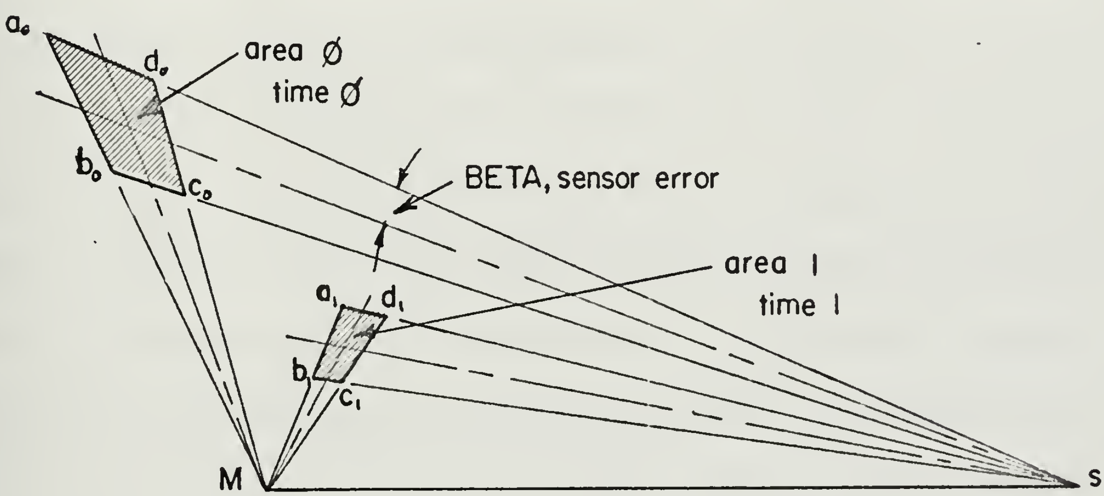
All these observations about feasibility ignore the question of cost except in that their concepts are simple applications of the state of the art; a cost analysis should be a separate study. However, since the computer might become a very expensive subsystem, certain efforts were made to simplify computation and memory requirements, and to open the door for analog computing where possible. These efforts are further detailed under the description of the model and the smoothing equations employed.

II. THE MODEL

A. THE SIMULATION APPROACH

Since a system simulation is not as satisfying as a more analytical study, a consideration of the conditions which prompted the simulation approach is in order.

A look at the system geometry is instructive.



From above, with a constant velocity target observed at two times, 0 and 1, the dot in the shaded area represents the true position of the target generating the bearings to the sensors, M and s. The bearing error at the sensor, assumed to be normally distributed about the true bearing with standard deviation σ, causes the target to appear to the system to be somewhere within the shaded area, but not uniformly distributed there. The apparent movement of the target from time 0 to 1 can be from any point in area 0 to any point in area 1, suggesting

a "worse case" study of targets by considering an apparent track which differs most from the true track, that is, targets moving from, say, corner b_o to corner d_1. But there is no guarantee that this sort of movement is really a "worst case" movement, since viewing the target as zigzagging from corner to opposite corner permits a simple averaging of the course vector to yield a useful value; perhaps the worst case is actually consistent movement throughout target run, from b_o toward d_n. In any case, the determination of what really is the limiting case provides only a rough tool for gauging system effectiveness. Since the target is not only not uniformly distributed in the area, but is also not distributed as a bivariate normal or as any other manageable distribution, any effort to calculate an apparent path and assign a probability to its occurrence seems doomed to failure. Furthermore, any technique used to smooth apparent target movement is irrevocably tied to the assumed nature of that movement and, unlike the case treated below in this paper, indicates no clear-cut best smoothing approach.

Several other problems arise with this approach. If the miss distance is taken as the measure of effectiveness, then the probabilities associated with various miss distances generated by the various tracks are known only if the track probabilities are known, as described above. Alternatively, taking the error areas (shaded in the above illustration) as error volumes in three dimensions formed by the intersection of conical solids, then projecting the error solid determined at the weapon firing time along the target course, we might calculate the volume of

intersection of the error volume and the weapon lethal volume, and express this result as a percentage of the error volume to give a probability of kill. Unfortunately, this kill probability is not a measure of system kill probability except in the complete kill and complete miss cases, because the target is not uniformly distributed in the error volume. Moreover, the study then hinges on the choice of firing decision rule which determines the position of the target at firing time, and thus the projected error volume.

Using a central limit theorem argument, the decision was made to sample target bearing from a normal distribution at discrete time intervals during target run, and smooth the target's position and velocity. In doing so, one gains insight into the functioning of the smoothing calculations in the real system and into the actual capabilities necessary for the system computer.

B. CONSTRUCTION OF THE COMPUTER MODEL

For ease of reference, the model description will follow the sequence of the enclosed computer program.

First, the system parameters are read in:

P1 - Time between recomputations of target position. This time represents the time between transmissions from s to M of the observed bearing to the target. The position of s relative to M is presumed to be updated every P1 seconds as well. P1 is entered in seconds.

OMEGA - Velocity smoothing constant corresponding to β in the α-β smoothing equations described below. $0 \leq$ OMEGA ≤ 1.

ALPHA - Position smoothing constant, the α of α-β smoothing equations. $0 \leq$ ALPHA ≤ 1.

BETA - The absolute value of the maximum error in a bearing observation by a sensor hydrophone. Since the error is normally distributed with mean equal to true bearing, and .9999 of the normal distribution lies within three standard deviations of the mean, BETA $\doteq 3\sigma$, where σ is the standard deviation of the error distribution. BETA is entered in degrees.

GAMMA - The corresponding bearing error in fixing s relative to M, also in degrees

DELTA - The range error in fixing s position; based on triggered pulse response or a one-way pulse timed in a system synchronized before launch, this should be very small. DELTA is entered in yards.

ZETA - Smoothing constant for Z, the computed vertical distance of target above weapon. $0 \leq$ ZETA ≤ 1.

TGTBG - Target bearing at start of run, from M. M is taken to be the origin of the cartesian coordinate system. TGTBG is entered in degrees, from north ≤ 360.

TGTRG - Initial target range from M, measured in yards.

TGTSPD - Target speed, assumed constant throughout the run, entered in knots.

TPASS - In lieu of target course, the target is "steered" so as to pass a set fraction of the distance from M to s. This fraction is TPASS, expressed as a decimal, $0 \leq \text{TPASS} \leq 1$.

TDPTH - Target depth, in feet.

WSPD - Weapon speed, in knots.

WDPTH - Weapon depth, maintained by a hydrostatic system and assumed by the program to be errorless. WDPTH is entered in feet.

SBRGM - True bearing of s from M, entered in degrees. Since we are considering only one slave with the master, arbitrarily limit $0 \leq \text{SBRGM} \leq 180$.

SRNGM - True range of s from M, in yards. In addition, all entries are REAL*4, i. e., single precision accuracy.

The program converts knots to yards per second, all bearings to radians, and places TGTBG between $-\pi$ and π radians. Depths are converted to yards and TPASS is converted to the x component of the value set in, to represent the x-axis intercept of the target. In cases in which s does not bear 090 degrees from M, this change will cause deviations of the track from that intended, but does not change the operation of the program or the results. Unless the model is modified to investigate target drift, s can always be taken as 090 from M, because in the single M-s pair, changing the target track has the same effect as changing the position of s relative to M, BETA1, GAMA1,

and DELTA1 are the corresponding errors expressed as standard deviations.

Target position is now converted to cartesian coordinates, and the target's course and speed are used to find the actual x and y components of velocity, and the distances in x and y which the target will move each P1 seconds. The z component of velocity is assumed to be zero.

Next the true vertical angle to the target and the cartesian coordinates of the slave sensor are computed. Target and slave bearings can then be obtained from subroutine GNRATE in a form suited to use in the program's particular form of the Law of Sines. The bearings to slave and target and range to the slave are modified by GAUSS, the normal distribution with the appropriate attributes, and the resultant bearings represent the bearings seen by the sensors. All calculations to this point have been for the use of the program alone; now the first system information is obtained.

The calculations of apparent target position are system calculations, as are the subsequent smoothing operations. Note that the program recycles after the first calculation of target position: the system needs two data points for first velocity input.

To obtain fire control outputs, the real-system computer finds the target track from the smoothed x and y velocity components, then solves for the intersection of that track with its normal through (0, 0), and treats the solution as the estimated CPA in the x-y plane. The true CPA will be slightly different in a case of large vertical component

of target velocity, consideration of which could be the basis of a later study, but there is still a strong argument for using the estimated CPA of this model for its ease of calculation, in spite of the slightly longer weapon run time that might be incurred. In this model, the weapon is aimed vertically with the vertical angle of that point having a horizontal range of the estimated CPA just found, and the current smoothed vertical position of the target. If vertical movement of the target becomes an important consideration, this model is oversimplified in this respect, and should employ α-β smoothing in the vertical plane just as is presently used in the horizontal plane.

When calculations are complete, the target is stepped to its position P1 seconds later, new bearing information is generated, and the calculation sequence is repeated until time of fire is less than P1 seconds away. The weapon is assumed to proceed to the estimated CPA point without error, and the miss distance is the difference between the estimated CPA and the target's true position projected to the estimated CPA time. Weapon run errors can be imposed on this program, though the additional complication of this step was deemed unnecessary at this stage, since the object of this model is to facilitate the study of sensor and computational systems.

One should note in passing that DMISS, the weapon miss distance, is not necessarily a measure of system effectiveness. DMISS is used as a measure of computational and smoothing accuracy; a system measure of effectiveness should involve the probability of passing close

enough to the target while en route to the estimated CPA to trigger a sensor. Furthermore, DMISS is measured strictly from the target noise source, treated as a point source; treating the targets as having finite volumes will greatly increase acceptable latitude in the size of DMISS.

No attention has been given in the model to such refinements as adjusting aim points for target volume, or adjusting the aim point for a surface target to be beyond the apparent target in order to take advantage of a weapon-borne proximity sensor.

C. COMPUTATIONAL CONSIDERATIONS

As indicated in the program description above, certain program segments parallel actual system calculations. The form of these calculations is closely tied to the computing capability of the real system, and hence system cost.

Calculating apparent target position by the Law of Sines using a digital computer requires careful treatment of the angles involved in order to use the composite statements of this program. Hence the seven logical and algebraic steps of this program segment also imply several preliminary logic steps before the target bearings are obtained. All these calculations can be performed as well by analog circuitry; moreover, the method of solution might be greatly simplified for an analog device since this is merely a triangulation problem.

The five steps used in a digital computation of CPA are a compact solution of two simultaneous equations (see Appendix B). Just as before,

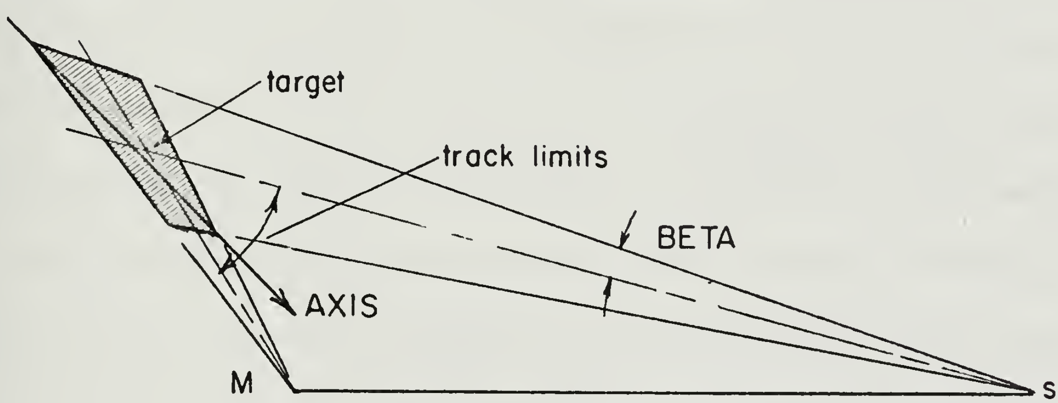
since this problem is just the solution of a right triangle, it might be very compactly and cheaply handled by an analog device.

Another problem encountered in the model points to another possible advantage of an analog computer over a digital. The nearly straight-on target, with x component of velocity near zero, has a track slope approaching infinity. The method used to handle this case in the model is to fix an upper limit on the slope, appropriate for the ranges under consideration, such that slopes in excess of that value imply directly that the projected x component of CPA is simply the last smoothed value of x position. A similar arbitrary cutoff is made for y component of velocity near zero. The problem is complicated by those targets very nearly zero degrees relative from (0, 0). Random errors in bearing observation cause the target to appear to be passing first between one pair of sensors, then another. This case is suppressed in the model, but implies a need in the system for suppressing one slave sensor's input or handling multiple inputs, and ensuring the capability to solve for CPA's of targets which will not pass between master and slave. An alternative solution to the problem, rather involved for a digital computer, is to have an alternate reference system available, rotated 45 degrees from the first, to handle the boundary-case targets. Certainly an analog system is readily adaptable to the slope-limit approach.

D. SMOOTHING EQUATIONS

Three basic alternatives were considered as methods of smoothing the apparent target track: least squares curve-fitting, moving averages, and exponential smoothing.

Under the assumption of a non-maneuvering target, a linear regression might be performed on the target's observed positions to obtain a smooth track line. Because of the large amount of noise in the bearing information, a large number of points is required to be assured of an adequate fit. The particular geometry of the problem, however, indicates that the number of sample points being fitted might be kept down since a large component of position error is along the target's true track. That is, looking at a typical "error area" within which the target is non-uniformly distributed,



we see that computed target position will favor the axis of the error area, and that this axis must bear an approximate alignment to target track, since the target always passes between M and s. Even if the required number of sample points can be kept small enough in this

case to keep computer memory requirements down, the problem is prohibitive with a maneuvering target. Fitting a second order curve to data points of high noise is senseless without a large number of sample points, because target position prediction can vary widely with even one outlier; if the large number of positions are obtained by too rapid sampling, apparent movement due to error exceeds true target movement, and nothing is gained; and long sampling periods to obtain sufficient yet well-spaced data require time on that order to adjust to new target movement, as well as large computing facilities. These considerations make pure regressions unacceptable for the model.

The principle of moving averages is simply a computing refinement of the regressions discussed above, because the method minimizes the least square error in the estimate of coefficients of the fitted curve. In this case, if we view the x and y components of target motion separately, and observe the changes in x and y during P1 as the values to be averaged, we can get a useful estimate of velocity. Unfortunately, a large number of points and therefore a large computer memory are required, and the rate of response is on the order of the time required to get those points. We might use this method, and we might apply a weighting method like Wiener's described in Reference 2 to handle the maneuvering target, but we would need relatively elaborate computing facilities.

These problems lead us directly to exponential smoothing, since this method requires the storage of only one previous data point per

component (first order smoothing), and has an adjustable response rate. Moreover, exponential smoothing can be readily performed by analog circuitry.

In this model, assuming no vertical component of velocity, smoothing in the x-y plane requires second order smoothing. Present position and velocity are smoothed and a prediction of future position is made, based on these values. The equations are the standard alpha-beta smoothing equations described in Appendix C.

Another important advantage accrues from the use of these smoothing equations. From the response curves of first order exponential smoothing showing the number of periods required to rise to 90% of a step input as a function of the smoothing constant, we can readily narrow the range of possible α's to be roughtly .18 to .4. Moreover, Benedict and Bordner have shown in Reference 1 that, for a ramp input, the optimal tracker is the optimal α-β tracker, and the optimal α-β tracker is obtained when

$$\beta = \alpha^2/(2-\alpha) \; .$$

Hence, insofar as a ramp input is descriptive of a maneuvering target, we are assured that our smoothing equations are optimal, and that only α need be simulation-tested over its possible range for optimality.

E. VERIFICATION OF THE MODEL

Since there is no existing system plausibly close to that being simulated, no validation of results was possible. Verification of the

model's performance was detailed, however, in that nearly every calculated value was printed at nearly every stage of the program, and checked against its inputs and against a graphical representation of the problem.

Specifically, the following measures were taken. The stepping constants which move the target each cycle were checked to see that they correspond to the x and y components of the velocity implied by target speed and TPASS. The true x and y positions of the target were observed to step the proper amount. The position of the slave sensor was checked in x and y coordinates to confirm the operation of function GNRATE. The time to CPA was calculated separately for the x and y components; that is, present position, component velocity, and CPA component were found consistent, and the implied times to CPS for the two components were found equal. Bearings at M and s were plotted and the resultant triangle was compared with the computed values of the angles at M, s and the target (ANGM, ANGS, ANGT) to ensure that the angles total 180 degrees, and that they tally with the input bearings. The change in size of each of the three angles was observed as the target progressed toward CPA, to ensure that changes were of the proper relative magnitude and direction, and to observe the effect of bearing errors on the progressing triangular solutions. The intercept BB was hand calculated from the velocity components, plotted and used with velocity to confirm CPS; the track intercept on the x axis was compared with TPASS.

The program was debugged with no GAUSS modification of bearings until all x and y observations concurred with true target position components, and DMISS was zero. GAUSS modification of the values was selectively introduced, and the effect of the smoothing on each stage of target movement was observed. To remove any possibility that multiple conversions of angles or chain dependent operations might degrade the accuracy of the results, the entire program was converted to double precision and rerun; no differences resulted. Finally, the target was stepped through 170 degrees to ensure that targets of varying position and track are properly handled.

One further note should be made. An initial discrepancy prompted the running of a test program to see what values are returned for sine, cosine, and arc tangent (ATAN) functions. Sine and cosine values were returned properly, but ATAN returned only first quadrant angles at the terminals under TSS, and returned first and second quadrant angles under OS/MVT. Where doubtful, the program now converts angles to first quadrant and reconverts to the proper quadrant when appropriate.

F. APPLICATION OF THE MODEL

The model was designed to facilitate a preliminary parametric and feasibility study of the described weapon system. To that end, several questions have evolved, some out of observation of the model, which are important to system design and to which the model should provide answers.

1. What is the effect of errors in base leg calculations, i.e., in placing s relative to M? Both bearing and ranging errors should be considered, and the results observed as a function of the distance of s from M and target track orientation relative to the system. The testing is performed in the model by replacing all "CALL GAUSS" statements except one (SBRNG, for example) by statements which bypass modification of the values (TBM=TGTBG, for example). Then GAMMA is stepped in increments while the targets are stepped and replicated in loops. The smoothing equations remove the independence of the sampling from step to step along the target's track, so a statistically viable result will require about ten replications per target track, depending on the confidence interval desired. Then a test of eight target starting positions for each of two TPASS values, at three values of s range from M, holding the range error constant and stepping the bearing error through ten increments implies 4800 replications or about one hour of computing time.

2. What is the effect of changes in the size of BETA, the target bearing sensing error? Again, the results are a function of base length and target track, and indicate about an hour of computer time for any fixed GAMMA and DELTA.

3. What is the effect of changes in target speed? The high and low speeds of the expected range are particularly important since a high-speed target may not provide sufficient smoothing time, and at low speeds the BETA error may obscure target movement. Hence

BETA, P1, and target speed are closely interdependent, and optimal smoothing may require adjusting P1 and/or ALPHA according to initial calculations of target speed.

4. If no P1 or ALPHA is adequate for all ranges of target speed, what values are optimal in which ranges, and what estimate of target speed is sufficient criterion for deciding to change P1 or ALPHA? Furthermore, should P1 or ALPHA be adjusted as target range decreases? The computer time required to answer questions 3 and 4 can only be estimated after defining the needs of the system (detection range and the expected BETA expected), and making broad category tests.

5. What is a general lower limit for weapon speed, based on expected target speeds? Roughly, target speed determines only the firing lead time required, and should simply be high enough that smoothing be adequate. Minimum weapon speed could be plotted as a function of target speed for a certain range of DMISS, for each set of system parameters considered. This would provide a clear picture of the payoff gained for the incremental cost of weapon speed changes.

6. Further study can be done of the effect of depth separation of weapon and target, but initially can be taken as 300 feet as a generally realistic value. The results are then the worst miss distances for a system floating at 300 feet, and the target between 0 and 600 feet.

III. CONCLUSIONS

1. Construction of a weapon system of the type described appears to be within the state of the art, and the needs of the system should be satisfied by analog circuitry.

2. Results indicate that the system provides sufficient accuracy to warrant further investigation. Preliminary runs with ALPHA = .4 and BETA less than five degrees yield DMISS ranging from 10 yards to 100 yards according to target track. Using a constructive target volume and the effects of explosives described in References 3 and 4, these values are within the lethal range of a small weapon.

APPENDIX A

Source level requirements for underwater sound transmissions - (All pages refer to Reference 6).

Consider the power losses in a transmitted sound wave:

Transmission loss (spreading) = 20 log r_2,

where r_2 is the distance traveled (pg. 83)

Absorption = about 20 db/kyd in seawater, at about

100 kHz (Fig. 5.2, pg. 88)

Then assuming we need 10 db at the receiver, at a range of 2000 yds., the power required is

10 db	receiver
66 db	transmission loss
40 db	absorption
116 db	transmitter

To produce 116 db, the transmitter needs (Fig. 4-3, pg. 63)

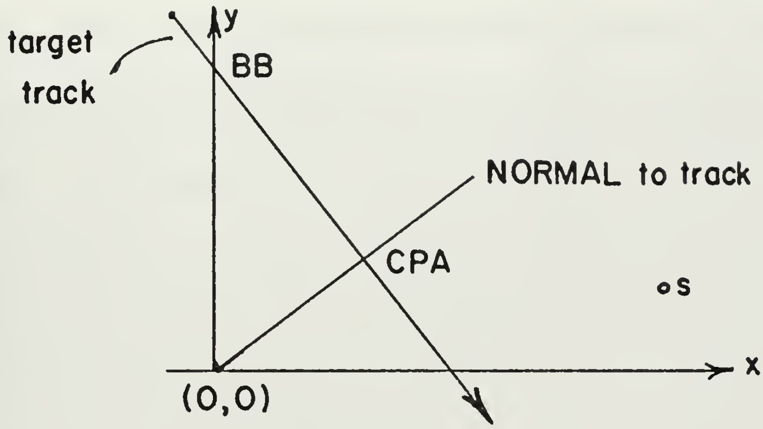
500 watts at DI = 20 db (DI = directivity index)

50 watts at DI = 30 db

A similar calculation at 3000 yds. shows 140 db required at the source, or 200 watts at DI = 40 db (extrapolated).

APPENDIX B

Derivation of equations for CPA solution.



Define: M = slope of target track

m = slope of normal to track

BB = y - axis intercept of track

Then the equation of the track line is

$$y = Mx + BB$$

And the equation of the normal to the track is

$$y = mx$$

Solving by substitution

$$mx = Mx + BB$$

whence

$$BB = x(m-M)$$

1) $x = BB/(m-M)$

2) $y = mBB/(m-M)$,

where BB is obtained by definition of M,

$$M = \frac{y - BB}{x}$$

or $$BB = y - Mx.$$

A check of the four applicable cases will show equations 1) and 2) are valid for slopes M defined as the ratio of y-component velocity to x-component velocity, properly signed, and regardless of whether the target has passed BB.

APPENDIX C

Second order exponential smoothing equations, " α - β ".

Exponential smoothing describes a linear combination of a predicted value and an observed value. Thus,

$$\overline{X}_i = \tilde{X}_i + (1 - \alpha)\hat{X}_i$$

where $\overline{X}_i$ = smoothed value

$\tilde{X}_i$ = rough input

$\hat{X}_i$ = predicted value,

by definition. Whence we can rewrite

$$(1)\ \overline{X}_i = \hat{X}_i + \alpha(\tilde{X}_i - \hat{X}_i).$$

Simultaneous analogous smoothing of velocity provides second order equations

$$(2)\ \overline{U}_i = \overline{U}_{i-1} + \beta/T\,(\tilde{X}_i - \hat{X}_i),$$

where the notation is as for (1), β is the weighting factor or smoothing constant analogous to α , and T is the time between observations. The predicted X_i is obtained from

$$(3)\ X_{i+1} = \overline{X}_i + T\overline{U}_i$$

```
      READ (5,9000)       P1
      READ (5,9000)         ALPHA
      READ (5,9000)           BETA
      READ (5,9000)             GAMMA
      READ (5,9000)                 DELTA
      READ (5,9000)     ZETA
      READ (5,9000)               TGTBG
      READ (5,9000)           TGTRG
      READ (5,9000)             TGTSPD
      READ (5,9000)           TPASS
      READ (5,9000)           TDPTH
      READ (5,9000)             WSPD
      READ (5,9000)                   WDPTH
      READ (5,9000)             SBRGM
      READ (5,9000)           SRNGM
 9000 FORMAT (F10.1)
      WRITE (6,9001)  P1,ALPHA,BETA,GAMMA,DELTA,TGTBG,TGTRG,
     1TGTSPD,TPASS,TDPTH,WSPD,WDPTH,SBRGM,SRNGM,ZETA
 9001 FORMAT (//,5X,'INPUT CHECK. . .',//,5X,'P1=',F7.1,2X,
     1'ALPHA=',F7.1,2X,'BETA=',F7.1,2X,'GAMMA=',F7.1,2X,'
     1DELTA=',F7.1,2X,'TGTBG',F7.1,//,5X,'TGTRG=',F7.1,2X,'T
     1F7.1,2X,'TPASS=',F7.1,2X,'TDPTH',F7.1,2X,'FEET',2X,'WS
     1F7.1,2X,'WDPTH=',F7.1,2X,'FEET',//,5X,'SBRGM=',F7.1,2X
     1F7.1,2X,'ZETA=',F7.1)
C     NOTE THAT SBRGM MUST BE LESS THAN 180 DEGREES
C     IN THIS ONE-SIDED SYSTEM
C                         * * *
C     SET IN CONSTANTS, INITIALIZE CLOCK AND COUNTER AND
C     MAKE RADIAN CONVERSIONS
      PI=3.141593
      IX=78133
      VC=.0174533
      CV=1.0/VC
      CLOCK=0.0
      NTALLY=0101
      CUTOFF=180.0*VC
      SAVE=TGTBG
      TGTBG=TGTBG*VC
      IF (TGTBG.LE.CUTOFF) GO TO 10
      TGTBG=TGTBG-2.0*PI
 10   SBGM=SBRGM*VC
      TPASS=TPASS*SRNGM*SIN(SBGM)
      TDPTH=TDPTH/3.0
      WDPTH=WDPTH/3.0
      BETA1=(BETA/3.0)*VC
      GAMA1=(GAMMA/3.0)*VC
      DELTA1=DELTA/3.0
      XREFM=0.0
      YREFM=0.0
C                         * * *
C     PROCEED TO CONVERT TARGET POSITION TO
C     CARTESIAN COORDINATES
      OMEGA=ALPHA**2/(2.0-ALPHA)
C     THIS VALUE CF OMEGA PRESUPPOSES A TARGET OF A
C     RAMP INPUT TYPE.  IF TARGET CHARACTERISTICS
C     ARE NON-MANEUVERING, RESULTS CAN BE IMPROVED
C     BY SETTING ALPHA=OMEGA=0.2, WITH  SOME LATITUDE
C     IN THESE VALUES
                          * * *
 100  TRUX=TGTRG*SIN(TGTBG)
      TRUY=TGTRG*COS(TGTBG)
      TRUZ=WDPTH-TDPTH
C                         * * *
C     COMPUTE STEPPING CONSTANTS FROM TARGET VELOCITY
      B=TPASS-TRUX
      ANGLE=ATAN(B/TRUY)
      SPEED=TGTSPD*(6076.115/(3.0*3600.0))
      XDOT=SPEED*SIN(ANGLE)
      YDOT=-(SPEED*COS(ANGLE))
      DELX=XDOT*P1
      DELY=YDOT*P1
C                         * * *
```

```
C      BEGIN THE ITERATED PROGRAM BY FINDING THE TRUE TARGET
C      ELEVATION AT M
 101   TRUELM=SIGN(ATAN(TRUZ/SQRT(TRUX**2+TRUY**2)),TRUZ)
C                          * * *
C      LOCATE ACTUAL POSITION OF S RELATIVE TO M
C      AND TARGET RELATIVE TO S
       IF (SBGM.GT.PI/2.0) GO TO 202
       SPOSX=SRNGM*SIN(SBGM)
       SPOSY=SRNGM*COS(SBGM)
       GO TO 203
 202   SPOSX=SRNGM*COS(SBGM-PI/2.0)
       SPOSY=-SRNGM*SIN(SBGM-PI/2.0)
 203   CALL GNRATE(TRUX,TRUY,SPOSX,SPOSY,TRUBGS)
C                          * * *
C      MODIFY THE TRUE BEARINGS TO THE TARGET AND THE BEARING
C      AND RANGE TO S BY SUBROUTINE GAUSS
C      SUBROUTINE GAUSS MODIFIES THE BEARING OR
C      RANGE WITH A NORMAL DISTRIBUTION WHOSE RANGE IS
C      +/- BETA (SAY), OR 3 STANDARD DEVIATIONS
       CALL GAUSS (IX,GAMA1,SBGM,SBRNG)
       CALL GAUSS  (IX,DELTA1,SRNGM,SRANGE)
       CALL GAUSS (IX,BETA1,TGTBG,TBM)
       CALL GAUSS (IX,BETA1,TRUBGS,TBS)
       CALL GAUSS (IX,BETA1,TRUELM,TELVM)
C                          * * *
C      USE THESE MODIFIED VALUES TO CALCULATE THE OBSERVED
C      POSITION OF THE TARGET BY THE LAW OF SINES
       ANGM=ABS(SBRNG-TBM)
       IF (TBS.GT.0.0) GO TO 301
       ANGS=ABS((SBRNG-PI)-TBS)
       GO TO 302
 301   ANGS=TBS+ABS(SBRNG-PI)
 302   ANGT=(PI-ANGM)-ANGS
       CALCRG=SIN(ANGS)*(SRANGE/SIN(ANGT))
C                          * * *
C      NOW TRANSLATE THIS RESULT INTO CARTESIAN COORDINATES
       XRUF=CALCRG*SIN(TBM)
       YRUF=CALCRG*COS(TBM)
C                          * * *
C      NOW OBTAIN THE SMOOTH VALUES OF X AND Y, AND USE THEM
C      TO CALCULATE THE ROUGH AND SMOOTH VALUES OF Z
C      THIS SET OF EQUATIONS REPRESENTS THE R-C ANALOG CIRCUI
C      FOR FIRST-ORDER SMOOTHING
C                          * * *
       PP1=P1-2.0
       PP2=P1+2.0
       IF (CLOCK.LT.PP1) GO TO 402
       IF (CLOCK.GT.PP2) GO TO 401
       UBARX=(XRUF-XSMTH)/P1
       UBARY=(YRUF-YSMTH)/P1
       UBARZ=0.0
       XPRED=XRUF
       YPRED=YRUF
       ZPRED=ZRUF
       ZSMTH=ZRUF
 401   XSMTH=XPRED+ALPHA*(XRUF-XPRED)
       UBARX=UBARX+(OMEGA/P1)*(XRUF-XPRED)
       IF (TRUX.LE.0.0.AND.UBARX.LE.0.0) UBARX=0.0
C      THIS EXCLUDES TARGETS NOT IN OUR ONE-SIDED SYSTEM
C                          * * *
       IF (ABS(UBARX).LT.30.0) GO TO 4011
       UBARX=SIGN(30.0,UBARX)
 4011  XPRED=XSMTH+P1*UBARX
       YSMTH=YPRED+ALPHA*(YRUF-YPRED)
       UBARY=UBARY+(OMEGA/P1)*(YRUF-YPRED)
       IF (UBARY.GT.0.0) UBARY=0.0
C      THIS EXCLUDES OUTBOUND TARGETS
C                          * * *
       IF (ABS(UBARY).GT.30.0) UBARY=-30.0
       YPRED=YSMTH+P1*UBARY
       GO TO 403
 402   XSMTH=XRUF
```

```
      YSMTH=YRUF
 403  ZRUF=TAN(TELVM)*SQRT(XSMTH**2+YSMTH**2)
      IF (CLOCK.LT.PP1) GO TO 601
C                        * * *
C     RECONVERT ANGLES TO DEGREES
      TBM=TBM*CV
      TBS=TBS*CV
      ANGM=ANGM*CV
      ANGS=ANGS*CV
      ANGT=ANGT*CV
      TELVM=TELVM*CV
      IF (CLOCK.LT.(30.0*P1)) GO TO 601
C                        * * *
C     THIS SECTION COMPUTES TARGET COURSE LINE AND IMPACT PO
C     REPRESENTING ANALOG COMPUTATION WITHIN THE SYSTEM
C                        * * *
C     FIRST TEST FOR THE BOUNDARY CASES (POINT,ZERO/INFINITE
      IF (ABS(UBARX).LT.0.001.AND.ABS(UBARY).LT.0.001) GO TO
      IF (ABS(UBARX).GT.ABS(UBARY/60.0)) GO TO 503
      CPAX=XSMTH
      CPAY=0.0
      CPATIM=ABS(CPAY-YSMTH)/ABS(UBARY)
      GO TO 599
C                        * * *
 503  IF (ABS(UBARY).GT.ABS(UBARX/60.0)) GO TO 504
      CPAY=YSMTH
      CPAX=0.0
      GO TO 505
C                        * * *
C     NEXT DEAL WITH INTERMEDIATE SLOPE TRACKS
 504  SLOPE=UBARY/UBARX
      BB=YSMTH-SLOPE*XSMTH
      ANORM=-(1.0/SLOPE)
      CPAX=BB/(ANORM-SLOPE)
      CPAY=ANORM*CPAX
      IF (ABS(UBARX).LT.0.001) GO TO 599
 505  CPATIM=ABS(CPAX-XSMTH)/ABS(UBARX)
 599  ZSMTH=ZETA*ZRUF+(1.0-ZETA)*ZSMTH
      CPAZ=ZSMTH
      FIRRNG=SQRT(CPAX**2+CPAY**2+CPAZ**2)
      CALL GNRATE (CPAX,CPAY,XREFM,YREFM,FIRBRG)
      IF (FIRBRG.GT.0.0) GO TO 500
      FIRBRG=2.0*PI-ABS(FIRBRG)
 500  FIRAZM=ARSIN(CPAZ/FIRRNG)
 501  RUNTIM=FIRRNG/WSPD
      CPACLK=CPATIM+CLOCK
      FIRCLK=CPACLK-RUNTIM
      FINX=TRUX+XDOT*CPATIM
      FINY=TRUY+YDOT*CPATIM
C                        * * *
C     COMPARE TARGET DETONATION COORDINATES (CPA) WITH FINAL
C     POSITION COORDINATES (FIN) TO OBTAIN MISS DISTANCE
      DMISS=SQRT((FINX-CPAX)**2+(FINY-CPAY)**2+(TRUZ-CPAZ)**
C                        * * *
C     CONVERT REMAINING ANGLES TO DEGREES
      FIRBRG=FIRBRG*CV
      FIRAZM=FIRAZM*CV
      WRITE (6,2005) CPATIM,CPAX,CPAY,CPAZ,FINX,FINY
 2005 FORMAT  (/,5X,'CPATIM=',F7.2,1X,'SECONDS',2X,'CPAX=',F
     1'CPAY=',F10.3,4X,'CPAZ=',F10.3,4X,'FINX=',F10.3,4X,'FI
      WRITE (6,2006) FIRRNG,FIRBRG,FIRAZM
 2006 FORMAT  (/,5X,'FIRRNG=',F10.3,4X,'FIRBRG=',F6.1,4X,'FI
      WRITE (6,2007) DMISS
 2007 FORMAT (///,5X,'DIS MISS IS DE MUTHAH....DMISS=',F10.3
C                        * * *
      C1=CLOCK+P1
      IF (FIRCLK.LT.C1) GO TO 602
C                        * * *
C     STEP CLOCK AND TARGET/RUN COUNT, AND UPDATE
C     TRUE TARGET BEARING
 601  CLOCK=CLOCK+P1
      NTALLY=NTALLY+1
```

```
      TRUX=TRUX+DELX
      TRUY=TRUY+DELY
      CALL GNRATE (TRUX,TRUY,XREFM,YREFM,TGTBG)
      GO TO 101
C                     * * *
 602  CLOCK=0.0
      MARK=MOD(NTALLY,100)
      NTALLY=(NTALLY+100)-(MARK-1)
      SAVE=SAVE+60.0
      IF (SAVE.GT.360.0) SAVE=SAVE-360.0
      TGTBG=SAVE*VC
      IF (TGTBG.GT.CUTOFF) TGTBG=TGTBG-2.0*PI
      WRITE (6,1111) SAVE
 1111 FORMAT (//,5X,'TGTBG FOR THIS RUN =',F7.1)
      IF (SAVE.GT.270.0.OR.SAVE.LT.070.0) GO TO 100
C                     * * *
      STOP
      END


      SUBROUTINE RANDU(IX,IY,YFL)
      IY=IX*65539
      IF(IY)5,6,6
 5    IY=IY+2147483647+1
 6    YFL=IY
      YFL=YFL*.4656613E-9
      RETURN
      END


      SUBROUTINE GAUSS(IX,S,AM,V)
      A=0.0
      DO 50 I=1,12
      CALL RANDU(IX,IY,Y)
      IX=IY
 50   A=A+Y
      V=(A-6.0)*S+AM
      RETURN
      END


      SUBROUTINE GNRATE(X,Y,XREF,YREF,BRG)
C     THIS SUBROUTINE ACCEPTS CARTESIAN INPUTS AND
C     RETURNS BEARINGS MEASURED +/- FROM NORTH, FROM
C     THE REFERENCE POSITION
      PI=3.141593
      AAB=ABS((X-XREF)/3500.0)
      IF(ABS(Y-YREF).GT.AAB) GO TO 1004
      AAA=3500.0
      GO TO 1005
 1004 AAA=ABS((X-XREF)/(Y-YREF))
 1005 BRG=ATAN(AAA)
      IF((Y-YREF).LT.0.0) GO TO 1010
      GO TO 1020
 1010 BRG=PI-BRG
 1020 IF((X-XREF).GE.0.0) GO TO 1030
      BRG=-BRG
 1030 RETURN
      END
```

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13. ABSTRACT

This paper is a response to a proposal before the Mine Advisory Committee that there be an analytically supported development of a modern explosive mining capability for the U. S. Navy. A proposed system is described, and a model is developed to provide the means to revaluate the system in comparison with other alternatives.

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